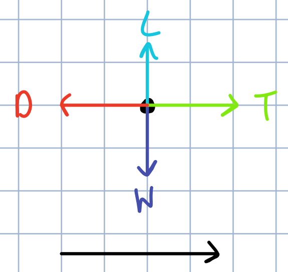
Treating an aircraft as a point mass :



for an aircraft in equalibrium $L = W$

$$\longrightarrow \quad C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = \frac{W}{\frac{1}{2}\rho V^2 S} = \frac{w}{q}$$

w → wing loading $\frac{W}{S}$

q → dynamic pressure $\frac{1}{2}\rho V^2$

The corresponding flight speed is therefore :

$$V = \sqrt{\frac{w}{\frac{1}{2}\rho C_L}}$$

∴ ↓V from ↓w (lighter / bigger wing)
↑C_L (↑a)
↑ρ

Minimum possible level flight speed occurs at $C_{L_{max}}$ (stall)

Equivalent Air Speed :

True air speed (TAS) V or V_T is speed relative to air mass in which its flying

Equivalent air speed is the speed at standard sea level density that gives the same aerodynamic loads.

$$V_E = V\sqrt{\frac{\rho}{\rho_0}} = V\sqrt{\sigma}$$

EAS is close to indicated air speed (IAS), measured via a pitot-static tube to measure q.

$$p_{pitot} - p_{static} = \frac{1}{2}\rho V^2 = \frac{1}{2}\rho_0 V_E^2$$

Bernoulli

Stall will always occur at same EAS and structural limits defined in terms of EAS.

Drag:

$D = T$ in eqlm $\therefore$ $C_D = \dfrac{T}{\frac{1}{2}\rho V^2 S}$

There are many additional sources of drag:

- Profile (skin + form)
- Interference: between components

....

Drag Equation

upper case = 30
lower case = 20

- Drag consists of zero-incidence coefficient C_{D_0}
- part $\propto C_L^2$
  - ↳ induced drag + form drag ⟶ drag due to lift

$$C_D = C_{D_0} + KC_L^2$$

$$K = \frac{1}{\pi e AR}$$

Also written as $$C_D = C_{D_0} + \frac{C_L^2}{\pi e AR}$$

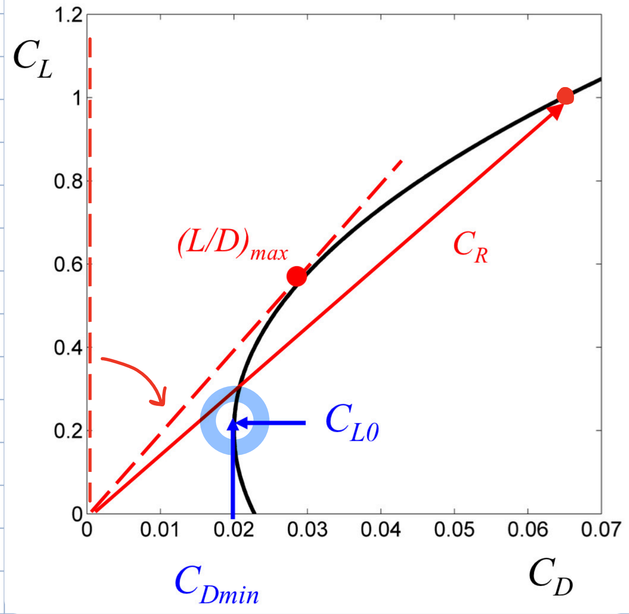
where e is redefined from span efficiency factor for wing to Oswald efficiency factor for the whole aircraft.

Drag Polar:

- For an aircraft with a cambered wing, minimum drag no longer necessarily occurs at 0 lift
  - ↳ (we will assume $C_{D_{min}} = C_{L_0}$ for year 1)

⟶ drag equation becomes $$C_D = C_{D_{min}} + K(C_L - C_{L_0})^2$$

Sometimes plotted as C_L vs. C_D



L/D max is tangent passing through origin

A line from origin to any point on graph is the vector of the aero force at that point. (C_R)

Drag Equation Expanded :

$$C_D = C_{D_0} + KC_L^2$$

$$C_L = \frac{W}{\frac{1}{2}\rho V^2 S}$$

$$\longrightarrow \quad C_D = C_{D_0} + \frac{KW^2}{(\frac{1}{2}\rho V^2 S)^2}$$

$$D = C_{D_0}\frac{1}{2}\rho V^2 S + \frac{KW^2}{\frac{1}{2}\rho V^2 S}$$

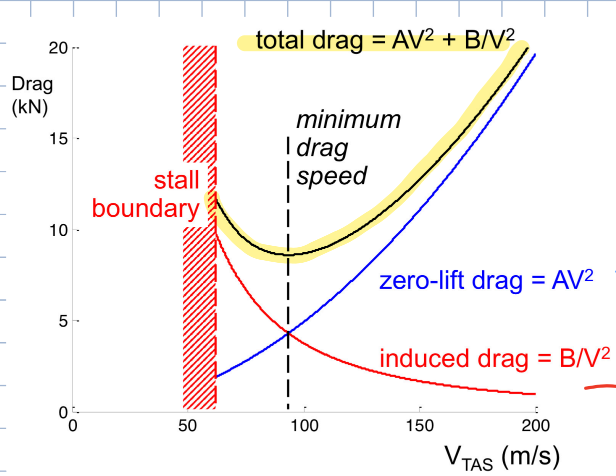
$$D = AV^2 + \frac{B}{V^2}$$

where A is 0 lift drag term, $A = C_{D_0}\frac{1}{2}\rho S$

and B is induced drag term, $B = \frac{KW^2}{\frac{1}{2}\rho S}$

} functions of alt. & ρ

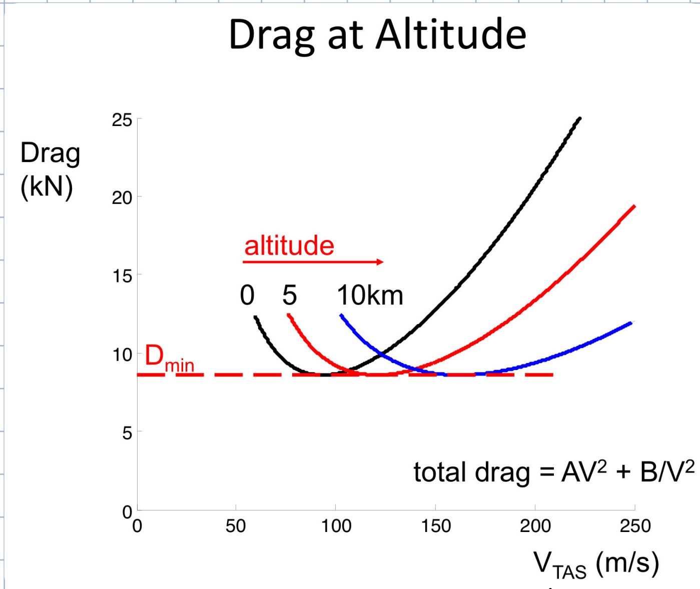
Drag as function of velocity : (at SL)



$\uparrow$ as $V\uparrow$: at high speed term A dominates

$\downarrow$ as $V\uparrow$: at low speed B dominates

Drag changing w/r to altitude :



As altitude ↑, ρ ↓ and therefore we must fly faster (V↑) to achieve steady level flight.

Alt. ↑, ρ ↓, V ↑

D_{min} doesn't change as V & ρ both affect drag and balance with changing altitude.

→ if plotted in terms of EAS, variation with altitude dissapears

→ one curve describes aircraft drag vs. V rather than many with changing alt.

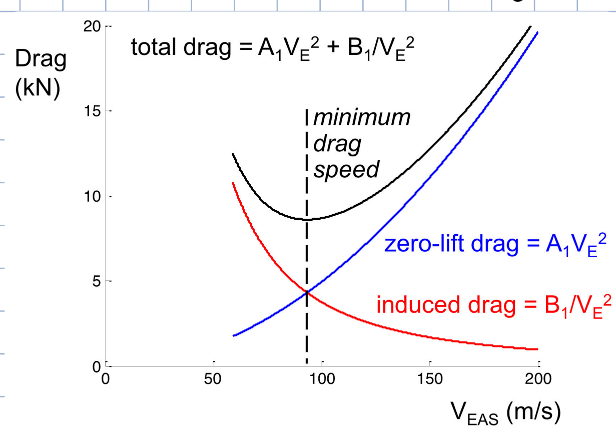
Drag equation in terms of EAS :

$$D = A_1 V_E^2 + \frac{B_1}{V_E^2}$$

where $A_1 = C_{D_0} \frac{1}{2} \rho_0 S$

and $B_1 = \frac{KW^2}{\frac{1}{2}\rho_0 S}$

constants as ρ constant (ρ_{SL})



Minimum Drag :

$$D = L \times \left(\frac{D}{L}\right) = W \times \left(\frac{C_D}{C_L}\right)$$

∴ min drag at min $\boxed{\frac{C_D}{C_L}}$

$$\frac{C_D}{C_L} = \frac{C_{D_0} + KC_L^2}{C_L} = \frac{C_{D_0}}{C_L} + KC_L$$

$$\frac{d(C_D/C_L)}{dC_L} = -\frac{C_{D_0}}{C_L^2} + K$$

at min where $= 0$, $C_{D_0} = KC_L^2$

$\therefore$ $C_{D_{min}} = 2C_{D_0} = 2KC_L^2$

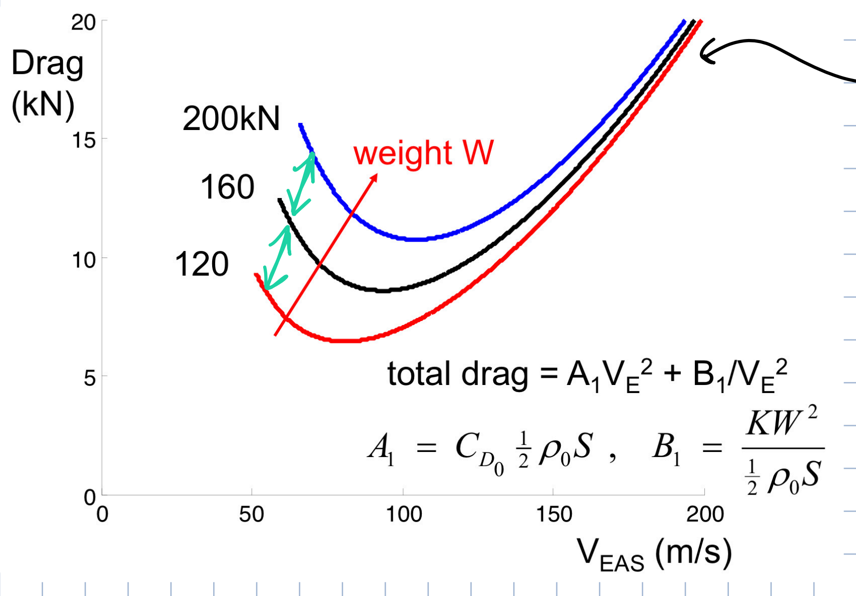
$$\left(\frac{C_D}{C_L}\right)_{min} = 2\sqrt{C_{D_0}K}$$

$$C_{L_{MD}} = \sqrt{\frac{C_{D_0}}{K}}$$

Subbing into V equation, $V = \sqrt{\frac{W}{\frac{1}{2}\rho S C_L}}$

$$\rightarrow \quad V_{MD} = \left(\frac{2W}{\rho S}\right)^{\frac{1}{2}}\left(\frac{K}{C_{D_0}}\right)^{\frac{1}{4}}$$

Effect of Weight on Drag:



at ↑V induced drag B becomes less significant

B includes W term

So disparity at ↑V becomes small

whereas at low speed B dominates

→ large difference